## Basic Algebra Skills Needed for Differentiating

In section 3-1, we started building a library of derivative rules and shortcuts. Here are the first few rules (when listed below $c$ is assumed to be a constant):

| Rule | Example |
|---|---|
| $\frac{d}{dx}(c) = 0$ | $\frac{d}{dx}(5) = 0$ |
| $\frac{d}{dx}(x^n) = nx^{n-1}$ | $\frac{d}{dx}\left(x^{3/5}\right) = \frac{3}{5}x^{-2/5}$ |
| $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$ | $\frac{d}{dx}(4x^3) = 4\frac{d}{dx}(x^3) = 12x^2$ |
| $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$ | $\frac{d}{dx}(x^{2.5} + x^{-4}) = 2.5x^{1.5} - 4x^{-5}$ |
| $\frac{d}{dx}(a^x) = a^x \ln(a)$ | $\frac{d}{dx}(2^x) = 2^x \ln(2)$ and $\frac{d}{dx}(e^x) = e^x$ |

In order to use the rules above, you need to be well practiced at working with fractions and exponents. Here is a quick review

| Rule | Example | Comment |
|---|---|---|
| $\frac{a}{b} = a \cdot \frac{1}{b} = \frac{1}{b} \cdot a$ | $\frac{5}{4} = 5 \cdot \frac{1}{4}$ and $\frac{3x^2}{4} = \frac{3}{4} \cdot x^2$ | Don't write mixed fractions in this class!<br>$5.25 = 5 + \frac{1}{4}$ is not the same as $5 \cdot \frac{1}{4} = 1.25$ |
| $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ | $\frac{2}{3}\frac{5}{7} = \frac{10}{21}$ and $\frac{x^3}{4}\frac{9}{x} = \frac{9x^2}{4}$ | You should be fast at simplifying products! |
| $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ | $\frac{3}{2} + \frac{2}{5} = \frac{15+4}{10} = \frac{19}{10}$ | Use whatever adding fraction method you know. |
| $x^a \cdot x^b = x^{a+b}$ | $x^4 \cdot x^{0.5} = x^{4.5}$ | Remember you can just count!<br>$x^2 \cdot x^4 = (x \cdot x) \cdot (x \cdot x \cdot x \cdot x) = x^6$ (I count 6). |
| $\frac{x^a}{x^b} = x^{a-b}$ | $\frac{4x^9}{5x^2} = \frac{4}{5}\frac{x^9}{x^2} = \frac{4}{5}x^7$ | Again you can just count as well<br>$\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x^3$ (I counted 3 after cancelling). |
| $\frac{1}{x^a} = x^{-a}$ | $\frac{3x^5}{2} \cdot \frac{11}{x^7} = \frac{33}{2}\frac{x^5}{x^7} = \frac{33}{2}\frac{1}{x^2} = \frac{33}{2}x^{-2}$ | In particular $\frac{1}{x} = x^{-1}$. |
| $\sqrt[b]{x^a} = x^{a/b}$ | $8^{2/3} = (8^{1/3})^2 = 2^2 = 4$ | Note that $\sqrt{x} = x^{1/2}$. |

In particular, if you had to take the derivative of any of the functions below, you would first have to expand and rewrite each term in the form $???x^{???}$. Try these for practice and talk to me if you are at all unsure about your work.

1. $\frac{2}{x^7}$
2. $x^{14} \cdot \frac{x^2}{3}$
3. $\frac{5}{x} \cdot \frac{x^7}{4}$
4. $\frac{1}{x^3} \cdot \frac{x}{4}$
5. $(\sqrt[3]{x} + 4)^2$
6. $3\sqrt[5]{x^7}$
7. $\sqrt{x}(2 - x)$
8. $\frac{100}{\sqrt{x}}$
9. $x^2(\sqrt{x} + 2)$
10. $\frac{1}{6} + \frac{2}{7}$
11. $\frac{x}{3}$
12. $\frac{15}{x^2}$